MAPLE

A Brief Primer

By

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This document is **NOT** a tutorial. In the tutorials that have been presented, the writer assumes that no knowledge of the subject exits. Then, the author describes the software in great detail including all of the intermediate steps that are required to learn and use that program. This will not be done herein.

In this primer the basic description of **MAPLE** is presented. This is then followed by a brief description on the syntax and the basic **usage** of this program. This document will then present examples of the important uses of **MAPLE** with little or no explanation. For a full presentation of this subject the reader is encouraged to read the following guides.

1- **MAPLE V** – Release 5 – **LEARNING GUIDE**

2- **MAPLE V** – Release 5 – **PROGRAMMING GUIDE**

Both of these documents are supplied with the **MAPLE** software and can be obtained from

Waterloo Maple, Inc.

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Maple V is a Symbolic Computation System. This refers to **Maple V's** ability to manipulate information in a symbolic or algebraic manner (rather than a numerical manner). Conventional mathematical programs require numerical values for all of the variables. **Maple** does not. **Maple** manipulates the underlying symbols and expressions.

Therefore, you may use this program to obtain exact analytical solutions to many mathematical problems. Integrals, systems of equations, differential equations, and problems in linear algebra many be done in an easy and efficient manner using **Maple**. A large set of graphical routines is also available in **Maple** so that the solution of many problems may be visualized.

Maple V's extensive mathematical capabilities are easily accessed through it advanced worksheet-based graphical interface. This interface is mouse driven. Standard operations for opening worksheets and saving documents are similar to those that are employed in most word processors.

At the top of the window, that is opened when the user “double clicks” on the **Maple** icon, is a **Tool Bar** that contains button-based shortcuts to common operations. Immediately below the tool bar is the **context bar** that contains controls that are specific to the task that is being currently used.

Below the context bar is a large area that displays the worksheet and is the region in which the user works.

In addition to **Maple** commands and their results, the user can include many other types of information such as

1- paragraphs of text

2- mathematical expressions and **Maple** commands within the same paragraph.

Some Useful Commands

I. Integer Computations

abs	absolute value of an expression
factorial	factorial of an integer
iquo	quotient of integer division
irem	remainder of integer division
iroot	roots of integers
isqrt	square root of integers
max,min	maximum and minimum of a set of inputs
mod	modulo arithmetic
surd	finding real roots
ifactor	factoring integers
isprime	tests for a prime number

II. Mathematical Functions

Sin,cos,tan,etc.	trigonometric functions
Sinh,cosh,tanh,etc.	hyperbolic trigonometric functions
arcsin,arcos,arctan,etc.	Inverse trigonometric functions
exp	exponential functions
ln	natural logarithmic functions

log[10]	logarithmic function base 10
sqrt	algebraic square root function
round	round to the nearest integer
trunc	truncate to the integer part
frac	fractional part
BesselI,BesselJ,etc	Bessel functions
binomial	binomial function
erf,erfc	error, complementary error functions
Heaviside	Heaviside step function
Dirac	Dirac delta function
LegendreKc,etc	Legendre elliptic integrals
hypergeom	hypergeometric function

III. Expression Manipulation

simplify	applies simplification rules to an expression
factor	factors polynomials
expand	reverse of factor
convert	converts expressions between different forms
polynom	converts series to polynomials
exp,expln,expsincos	converts trigonometric expressions to exponential form
parfrac	converts rational expressions to partial fraction form

rational	converts floating point numbers to rational form
radians,degrees	converts degrees and radians
set,list,listlist	converts between data structures
normal	transforms rational expressions into factored normal form
combine	combines terms in sums, products, and powers into a single term
lhs	takes the left side of an expression
rhs	takes the right hand side of an expression
numer	takes the numerator of a rational expression
denom	takes the denominator of a rational expression
nops	tells the user how many parts to an expression
op	allows the user to access parts of an expression
eval	evaluates the results of an expression
solve	solves an expression for its variables
fsolve	the numerical equivalent of solve

IV. Working with Polynomials

coeff	extract coefficient
lcoeff	find leading coefficient
tcoeff	find trailing coefficient
coeffs	return a sequence of all coefficients
degree	determine the highest degree of the polynomial
ldegree	determine the lowest degree of the polynomial
content	content of a multivariate polynomial
compoly	polynomial decomposition
discrim	discriminant of a polynomial
gcd	greatest common divisor
interp	polynomial interpolation
lcm	lowest common multiple
norm	norm of a polynomial
prem	pseudo-remainder
primpart	primitive part of a multivariate polynomial
randpoly	random polynomial
recipoly	reciprocal polynomial
resultant	resultant of two polynomials
roots	roots over an algebraic number field
sqrfree	square-free factorization

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When you load **Maple**, it loads only the kernel that is the base of the **Maple** system. In order to use some of the packages that are available, the user must load these packages using

with(package);

Some of the packages that are available within **Maple** are:

algcurves	algebraic curve tools for studying one-dimensional curves that are defined by multi-variate polynomials.
codegen	tools for creating, manipulating, and translating **Maple** procedures into other languages.
combinat	combinational functions, including commands for calculating permutations and combinations of lists.
DEtools	tools for manipulating, solving, and plotting systems of differential equations.
difforms	commands for handling differential forms; for problems in differential geometry.

genfunc	commands for manipulating rational generating functions.
geom3d	commands for three-dimensional Euclidean geometry; to define and manipulate points, lines, planes, triangles, spheres, polyhedra, etc. in three dimensions.
geometry	commands for two-dimensional Euclidean geometry; to define and manipulate points, lines, triangles, and circles in two dimensions.
inttrans	commands for working with integral transforms and their inverses.
linalg	commands for matrix and vector manipulation.
Matlab	commands to use several of Matlab's numerical matrix functions, including eigenvalues and eigenvectors, determinants, and LU-decomposition. This is only accessible if Matlab is installed on the system.
numapprox	commands for calculating polynomial approximations to functions on a given interval.
PDEtools	tools for manipulating, solving and plotting partial differential equations.

plots	commands for different types of specialized plots, including contour plots.
plottools	commands for generating and manipulating graphical objects.

In the Appendix, are 39 examples of the use of some of the more useful **Maple** functions. Their **Maple** commands as well as the results are also presented**.** A brief explanation of each of these examples now follows. Note that each command in **Maple** must terminate with a **;**

Example 1 - Addition of two numbers.

Example 2 - The limit of a function as the variable approaches a certain value.

Example 3 - Solution of an algebraic equation for its roots.

Example 4 - Expansion of an expression.

Example 5 – Definite integral of an expression.

Example 6 – Computation of a complicated number.

Example 7 – Evaluation of the previous result. **Note that the use of % refers the computation back to the previous expression.** Thus for a series of computations, the user need **not** type in the results again**.**

Example 8 – Find the sum of a given series.

Example 9 – Rationalize a complex number and obtain its real and imaginary parts.

Example 10 – Convert the complex number of example 9 to its polar form. **Once again note the use of %.**

Example 11 – Evaluate a special symbol to any significant number of decimal places.

Example 12 – Create an expression.

Example 13 – Operate on the previous expression. In this case, expand that expression.

Example 14 – Factor the previous result. Obtain the original expression from Example 12.

Example 15 – Simplify a long and complex expression.

Example 16 – Simplify a quotient.

Example 17 – Create an expression for further manipulation.

Example 18 – Expand that expression and create a new expression for further use.

Example 19 – Evaluate the value of the expression of Example 18 at a specific point.

Example 20 – Use the expression from Example 18 as a numerator. Use a new and expanded function as a denominator. Obtain the quotient by the use of the **normal** command.

Example 21 – Create a new expression for further use.

Example 22 – Obtain the **partial fraction** representation of the expression from Example 21.

Example 23 – Create a function **(not the same thing as an expression).**

Example 24 – Evaluate the above function at a numerical point.

Example 25 – Evaluate the function of Example 23 at a **symbolic** point.

Example 26 – Create an equation and solve it for the unknown variable. **Note the use of the brackets {} within the expression.**

Example 27 – Create 4 simultaneous equations of 5 variables. Solve these equations for 4 of the variables in terms of the fifth. **Once again, note the use of the {} brackets.**

Example 28 – Verify the solution to the above equations. **Note the use of % to refer back to the previous result.**

Example 29 – Define a function that will be differentiated. **Note the use of -> which describes the function and the dependent variable.** Differentiate the function with respect to the independent variable. Obtain a symbolic representation of the differentiation. Evaluate the results of the

differentiation.

Example 30 – Integrate the results of Example 29. Obtain its value. Simplify the expression. This yields the original function of Example 29.

Example 31 – Obtain a definite integral of the results of Example 29. **Note the use of the limits.**

Example 32 – Find the limit of an expression as the variable approaches a specific value.

Example 33 – Create an expression. Approximate that expression by a series near a specific point.

Example 34 – **Before attempting this example, the user must invoke the tools for differential equations. Thus the command WITH (DEtools); must be issued.** Create a differential equation. **Note the use of the symbolic differential operator. Note the nesting of this operator for higher derivatives.** State the initial conditions. Solve this differential equation for the dependent variable.

Example 35 – **Before attempting the next few examples, the user must invoke the tools of linear algebra. This is done by issuing the command WITH (linalg); .** Create a 3X3 matrix.

Example 36 – Find the determinant of the matrix of Example 35.

Example 37 – Find the inverse of the above matrix.

Example 38 – Multiply two matrices together.

Example 39 – Define a new matrix. Obtain its eigenvalues and eigenvectors.

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Appendix

Examples of Maple

	Algebraic	Task	Maple Command	Maple Result
1	Add	1+1	1+1;	2
2	Limit of	$x-7, x\to 3$	limit (x-7, x=3);	-4
3	Solve for x	$ax^2 = 4$	solve (a*x^2=4,{x});	$\{x = 2\frac{1}{\sqrt{a}}\}, \{x = -2\frac{1}{\sqrt{a}}\}$
4	Expand	$(a+b)^3$	expand ((a+b)^3);	$a^3 + 3a^2b + 3ab^2 + b^3$
5	Integrate	$\int_0^\beta x^2 \sin(\alpha x)dx$	int(x^2*sin(alpha*x),x=0..beta);	$-\frac{\alpha^2\beta^2\cos(\alpha\beta)-2\cos(\alpha\beta)}{\alpha^3} - \frac{2\alpha\beta\sin(\alpha\beta)+2}{\alpha^3}$

6	**Compute**	$\frac{2^{30}\sqrt{3}}{3^{20}}$	**(2^30/3^20)*sqrt(3);**	$\frac{1073741824}{3486784401}\sqrt{3}$
7	**Evaluate 6**		**value(%);**	$\frac{5150805672\ 7594732913\ 722}{4062664893\ 8819200088\ 497}$
8	**Sum the series**	$\sum_{1}^{\infty}\frac{1}{k^{2}}$	**sum(1/k^2,k=1..infinity);**	$\frac{1}{6}\pi^{2}$
9	**Complex Number**	$\frac{3+5i}{7+4i}$	**(3+5*I)/(7+4*I);**	$\frac{41}{65}+\frac{23}{65}I$
10	**Convert 9 to Polar**		**convert(%, polar);**	$polar\left(\frac{1}{65}\sqrt{2210}, \arctan\left(\frac{23}{41}\right)\right)$

11	**Evaluate to 40 places**	e^{1}	**evalf(exp(1.0), 40);**	2.718281828 97757
12	**Create an expression**	$(x+y)^{15}$	**expr := (x+y)^15**	$\exp\ r = (x+y)^{15}$
13	**Expand example 12**		**expand(expr);**	$x^{15} + 15\,yx^{14} + 105\,y^{2}x^{13} + 455\,y^{3}x^{12} + \cdots + 15\,y^{14} + y^{15}$
14	**factor example 13**		**factor(%);**	$(x+y)^{15}$
15	**Simplify**	$\cos(x)^{5} + \sin(x)^{4} + 2\cos(x)^{2} - 2\sin(x)^{2} - \cos(2x)$	**simplify(cos(x)^5 + sin(x)^4 + 2*cos(x)^2 - 2*sin(x)^2 - cos(2*x));**	$\cos(x)^{5} + \cos(x)^{4}$

16	**Normalize**	$\frac{x^3 - y^3}{x^2 + x - y - y^2}$	**normal((x^3-y^3)/(x^2+x-y-y^2));**	$\frac{y^2 + xy + x^2}{y + 1 + x}$
17	**Create an expression**	$(41x^2 + x + 1)^2 (2x - 1)$	**expr1 :=(41*x^2+x+1)^2*(2*x-1);**	$\exp r1 = (41x^2 + x + 1)^2 (2x - 1)$
18	**Expand example 17**		**expr2 :=expand(expr1);**	$\exp r2 = 3362x^5 - 1517x^4 + 84x^3 - 79x^2 - 1$
19	**Evaluate 18 at x=1**		**eval(expr2 , x=1);**	**1849**

20	**Use previous calculation**	**top :=expr2;**	$top = 3362\,x^5 - 1517\,x^4 + 84\,x^3 - 79\,x^2 - 1$
		bottom :=expand((3*x+5)*(@*x-1));	$bottom = 6x^2 + 7x - 5$
		answer :=normal(top/bottom);	$answer = \frac{1681\,x^4 + 82\,x^3}{3x+5} + \frac{83\,x^2 + 2x + 1}{3x+5}$
21	**Create expression**	**my_expr := (a*x^2+b)/(x*(-3*x^2-x+4));**	$my\_expr = \frac{ax^2 + b}{x(-3x^2 - x + 4)}$
22	**Partial fraction of 21**	**convert(my-expr, parfrac, x);**	$\frac{1}{4}\frac{b}{x} - \frac{1}{28}\frac{16a + 9b}{3x+4} - \frac{1}{7}\frac{a+b}{x-1}$

23	**Create a function**	$x^2+\frac{1}{2}$	**f := x -> x^2+1/2;**	$f := x \to x^2+\frac{1}{2}$
24	**Evaluate 23 at x=2**		**f(2);**	$\frac{9}{2}$
25	**Evaluate f(a+b)**		**f(a+b);**	$(a+b)^2+\frac{1}{2}$
26	**Solve an equation for x**	$x^3-\frac{1}{2}ax+\frac{13}{3}x^2=\frac{13}{6}ax+\frac{10}{3}x-\frac{5}{3}a$	**eqn :=x^3-1/2*a*x+13/3*x^2= 13/6*a*x+10/3*x-5/3*a** **solve(eqn, {x});**	$x^3-\frac{1}{2}ax+\frac{13}{3}x^2=\frac{13}{6}ax+\frac{10}{3}x-\frac{5}{3}a$ **Try this Result**

27 Solve 4 simultaneous equations in 5 variables

$$\begin{aligned} a+2b+3c+4d+5e&=41\\ 5a+5b+4c+3d+2e&=20\\ 3b+4c-8d+2e&=125\\ a+b+c+d+e&=9 \end{aligned}$$

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eqn1 := a+2*b+3*c+4*d+5*e=41;
eqn2 := 5*a+5*b+4*c+3*d+2*e=20;
eqn3 := 3*b+4*c-8*d+2*e=125;
eqn4 := a+b+c+d+e=9;

solve( {eqn1, eqn2, eqn3, eqn4 },
{a, b, c, d} );
```

$$\left\{\begin{aligned} d&=-\frac{4}{13}e-\frac{79}{13},\\ b&=\frac{22}{13}e-\frac{313}{13},\\ c&=-\frac{31}{13}e+\frac{483}{13},\\ a&=2 \end{aligned}\right\}$$

28 Verify solution of equations 1 and 2

```
eval( {eqn1, eqn2} , %);
```

$$\{41=41, 20=20\}$$

29	**Define and differentiate a function**	**f :=x -> x*sin(a*x)+b*x^2;**	$f = x \rightarrow x\sin(ax) + bx^2$
		Diff (f(x), x);	$\frac{\partial}{\partial x}\left(x\sin(ax) + bx^2\right)$
		f_prime :=value(%);	$f\_prime = \sin(ax) + x\cos(ax)a + 2bx$
30	**Integrate the above function**	**Int (f_prime, x);**	$\int \sin(ax) + x\cos(ax)a + 2bx\,dx$
		value(%);	$-\frac{\cos(ax)}{a} + \frac{\cos(ax) + ax\sin(ax)}{a} + bx^2$
		simplify(%);	$x\sin(ax) + bx^2$

31	**Definite integral for example 30**	**Int(f_prime, x=1..2);**	$\int_1^2 \sin(ax) + x\cos(ax)a + 2bxdx$
		value(%);	$2\sin(2a) + 3b - \sin(a)$
32	**Limit of an expression**	**expr := ((2*x+3)/(7*x+5));**	$\exp r = \frac{2x+3}{7x+5}$
		Limit(expr, x=infinity);	$\lim_{x\to\infty} \frac{2x+3}{7x+5}$
		value(%);	$\frac{2}{7}$

33	**Approximate an expression by a series**	**expr := sin(4*x)*cos(x);**	$\exp r = \sin(4x)\cos(x)$
		approx1 := series (expr, x=0);	$approx1 = 4x - \frac{38}{3}x^3 + \frac{421}{30}x^5 + O(x^6)$
34	**Solve a differential equation with initial conditions**	**diff_eq1 := D(D(y))(x) + 5*D(y)(x)+6*y(x) =0;**	$diff\_eq1 = (D^2)(y)(x) + 5D(y)(x) + 6y(x) = 0$
		init_con := y(0)=0, D(y)(0)=1;	$init\_con = y(0) = 0, D(y)(0) = 1$
		desolve({diff_eq1, init_con} , {y(x)});	$y(x) = e^{(-2x)} - e^{(-3x)}$

35	**Define a matrix**	**A :=matrix(3,3, [1/2, -1/3, 2, -5, 14/3, 9, 0, 11, -5/6});**	$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{3} & 2 \\ -5 & \frac{14}{3} & 9 \\ 0 & 11 & -\frac{5}{6} \end{bmatrix}$
36	**Find the determinant of example 35**	**det(a);**	$-\frac{2881}{18}$

37	**Find the inverse of matrix A**	**inverse(A);**	$\begin{bmatrix} \frac{1852}{2881} & \frac{-391}{2881} & \frac{222}{2881} \\ \frac{75}{2881} & \frac{15}{2881} & \frac{261}{2881} \\ \frac{990}{2881} & \frac{99}{2881} & \frac{-12}{2881} \end{bmatrix}$
38	**Multiply A by A**	**C := multiply(A,A);**	$C = \begin{bmatrix} \frac{23}{12} & \frac{365}{18} & \frac{-11}{3} \\ \frac{-155}{6} & \frac{1102}{9} & \frac{49}{2} \\ -55 & \frac{253}{6} & \frac{3589}{36} \end{bmatrix}$

39	**Define a matrix**	**B := matrix(2,2, {1,2,2,3});**	$B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
	Obtain the eigenvalues	**eigenvalues(B);**	$2+\sqrt{5},\ 2-\sqrt{5}$
	Obtain the eigenvectors	**eigenvects(B);**	$\left[2+\sqrt{5}, 1, \left\{\left[1, \frac{1}{2}+\frac{1}{2}\sqrt{5}\right]\right\}\right],$ $\left[2-\sqrt{5}, 1, \left\{\left[1, \frac{1}{2}-\frac{1}{2}\sqrt{5}\right]\right\}\right]$